Speed Measurement by Optical Techniques

1 Introduction

The most obvious optical method for measuring the speed of a moving object is to use the Doppler effect: light reflected from the moving object will be shifted in frequency on proportion to the line of sight velocity of the object. While we are familiar with examples in the radio domain (such as police traffic radar and Doppler weather radar) and with acoustic waves (the apparent change in pitch of a passing siren), demonstrating the Doppler effect optically is not so trivial a matter. The primary reasons for this are:

1. Optical frequencies are much too high ($\approx 10^{14}$ Hz) to be measured directly, and
2. The shift corresponding to typical speeds is on the order of $0.1 - 10$ MHz, much less than the linewidth of a typical He-Ne laser.

It is nonetheless possible to demonstrate the Doppler effect interferometrically using simple (if not cheap) equipment. In this experiment, you will use a small He-Ne laser, an electronic spectrum analyzer, a photodiode detector, and some simple optical elements to measure the shift in frequency of light scattered from the back of a small moving train and from particle suspended in flowing water. From this information you will be able to determine the velocity profile of the water flowing through a tube, the speed of a train, the speed of a wheel, and (after directly measuring the wheel speed) the wavelength of the laser.

One disadvantage of the Doppler effect to measure speed is that you can only determine the velocity component along the line of sight. It is possible to measure the transverse speed optically in spite of this handicap, again interferometrically. The trick is to project first an interference pattern onto the object, then measure the frequency of the light fluctuations as it passes through the fringes. The fringe spacing is determined from the geometry of the experimental setup, and the speed is then given by the product of this spacing and the characteristic frequency of the scattered light fluctuations. For this part of the experiment a rotating wheel will again be used, although a much faster rotation rate is required for the effect to be readily observable.

2 Theory and Concepts

At this point the question arises "How do we get around the problems mentioned in the introduction?" The answer is a technique known as *heterodyne interferometry*. This method involves combining the frequency-shifted or modulated optical (or radio, microwave, etc.) signal with an unshifted reference signal. The so-called "square-law" detector (which includes all common optical detectors) will respond only to the intensity (and intensity variation) of the combined beam. If the frequencies of the two signals differ by less than the bandwidth of the detector and are not identical, the electrical output of the detector will be modulated at a frequency equal to the difference between the frequencies of the two beams. For simplicity let us assume two electromagnetic waves of equal amplitude A and frequencies ω_1 and ω_2. If the two waves are incident on a detector we measure a signal

$$S = \langle (A\cos\omega_1 t + A\cos\omega_2 t)^2 \rangle \tag{1}$$

where $\langle\rangle$ denotes averaging performed by the detector and subsequent electronics. Using the identity $2\cos\omega_1 t\cos\omega_2 t = \cos(\omega_1-\omega_2)t + \cos(\omega_1+\omega_2)t$ we can write the signal as

$$S = A^2\langle\cos^2\omega_1 t\rangle + A^2\langle\cos^2\omega_2 t\rangle + A^2\left\langle\cos(\omega_1-\omega_2)t\right\rangle + A^2\left\langle\cos(\omega_1+\omega_2)t\right\rangle \quad (2)$$

Typically, in particular if we work with light, the frequencies $\omega_{1,2}$ cannot be resolved directly and the detection system averages the terms 1,2, and 4 in Eq. (2). The time dependence of the signal S (third term in Eq. (2)) is then characterized by a frequency

$$\omega_{beat} = |\omega_1 - \omega_2|.$$

This modulation is also referred to as a *beat frequency* or a *beat note* after the analogous acoustic phenomenon[1].

So the heterodyne technique eliminates problem 1 above, as long as the Doppler shift is not too large. But what of problem 2? The stability of a small He-Ne laser, over a time span of a few seconds, is certainly no better than a several MHz. Thus you should not be able to measure speeds $\leq$ 2m/s $\approx$ 10 km/hr, right? Wrong, as can be seen by comparing the coherence time of such a laser (tens of ns), the time delay between the signal and reference beams for the experimental arrangements used here ($<$ 1 ns), and the Schawlow-Townes linewidth of a laser ($\approx$ Hertz). Qualitatively, what **would** be the limit of such a measurement? (Note: It is worth devoting a few lines to this in your report.)

The theory of the Doppler effect is covered in most basic physics texts, and will not be discussed here. The arrangement in the first part of this experiment is identical to a Michelson interferometer, and the expected frequency shift of the light can be derived by considering the object to be equivalent to the translating mirror. From this point of view, the beat note signal is just the rate at which fringes pass the detector. In the second (fast wheel) part, the measured frequency is the quotient of the transverse speed of the wheel and the fringe spacing, which is determined by the wavelength of the light used and the angle of intersection of the two beams. For the third (water flow) part of the experiment, the angle of intersection of the beams and the refractive index of water ($n \approx 1.33$) must be taken into consideration. The necessary derivations are left to the student and a summary thereof should be included in the lab writeup.

All three parts of this experiment involve measuring the light scattered from a large number of randomly placed scattering centers. Given that the characteristic frequency of interest is the same for all of the scatterers, the signal from each will be randomly phased so one might expect any modulation at that frequency to average out. Furthermore, since the collected light is due to scattering, the total intensity can vary wildly at frequencies unrelated to the one in which we are interested. Considering such a small signal to noise ratio, how can we expect to measure anything? (Hint: The net signal from an ensemble of randomly phased sources is a problem analogous to the drunkard's walk. Also, the beauty of using spectrum analysis is that it measures the total amplitude of signals over only a small range of frequencies. Thus the appropriate figure of merit is the ratio S/N where S is the signal amplitude at a frequency of interest and N is the amplitude of the noises whose frequencies are within one resolution bandwidth of the signal frequency.)

[1] When two nearly identical tones are played together, the result is a 'waa-waa-waa' variation of the loudness with a periodicity equal to the inverse of the frequency difference of the two tones.

3 Experimental Procedures

This experiment is divided into three parts: a) Doppler shift measurement of the speed of a train and a rotating wheel; b) Transverse velocity measurement of the speed of a different wheel using scattering from a fringe pattern; c) Doppler shift measurement of the velocity profile of water flowing through a tube.

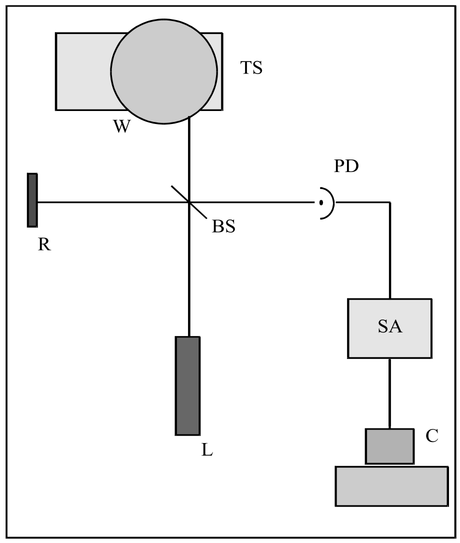

Figure 1: Experimental setup for the slow wheel Doppler Shift measurements. L is the laser source, BS is a beamsplitter, W is a motor driven wheel with reflective tape on its rim, R is a reflector (also tape covered), and PD is the optical detector. The electrical signal is sent to the 3585A spectrum analyzer SA for analysis and then to the computer C for recording. The translation stage TS is provided for positioning the wheel.

Doppler shift from a train and a rotating wheel. The experimental setup for the wheel is sketched in Fig. 1. Note the equivalence of this arrangement to a Michelson interferometer. The beam from laser L is split by beamsplitter BS and directed onto the edge of the slowly rotating wheel W and reference reflector R, both of which are covered with retroreflecting tape (of the type used by runners, bicyclists, etc., to avoid getting hit by cars). This tape is composed of small beads embedded in a substrate with a matched refractive index such that light entering the bead is refracted to the back of the bead where it is internally reflected back toward the source. This eliminates the need for the precise alignment required when flat mirrors are used. The retroreflected beams are then recombined at the beamsplitter and continue to detector PD. The electrical signal from PD is sent to a spectrum analyzer SA for measurement.

A spectrum analyzer (HP 3585A) is used to measure the beat frequency may be measured directly. You can transfer the spectra to a computer - see Appendix A for instructions. First, calculate the expected Doppler shift from an estimate of the wheel rotation rate and set the analyzer accordingly. Before connecting the detector signal to the spectrum analyzer, look at it with an oscilloscope; what is the amplitude of the signal, and does its period correspond to your calculated estimate? Measurements are to be made over a range of wheel positions at roughly one centimeter intervals, for which translation stage TS is provided. The range chosen should be such that one edge is accessible. Note: You must be sure that you are looking at the right peak on the spectrum analyzer trace. Determine this by blocking either the reference or shifted beams (the signal will then disappear) and by translating the wheel (the signal frequency should then shift).

Once the data are taken, you will also need the diameter of the wheel, a direct measurement of its rotation rate, and the wavelength of the laser. From these you should be able to determine the rotation rate of the wheel and to estimate the precision of your measurements. Discuss the precision of your results and the important error sources.

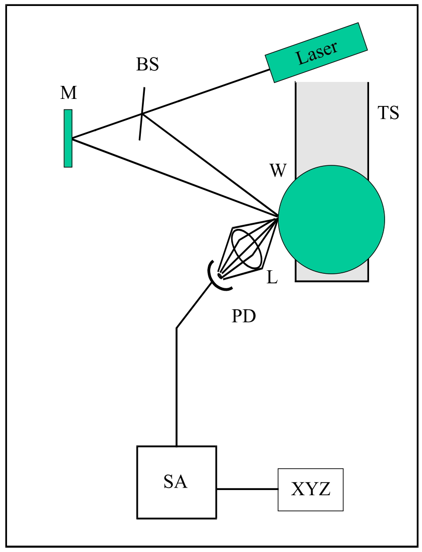

Figure 2: Experimental setup for the transverse speed measurements. Most labelled parts are as in Fig. 1, except that L is a collecting lens (optional) M is a flat mirror, and XYZ is a PC to which the spectra can be transferred.

Transverse speed measurement. As pointed out earlier, the Doppler technique is applicable only for measuring longitudinal velocities. (There is in fact a transverse Doppler effect, but it goes as v^2/c^2 and so is observable only for relativistic particles.) However, it is possible to measure the transverse velocity of an object using light scattering. The setup is as shown in Fig. 2. The laser beam is split and reflected by BS and mirror M onto the edge of the fast rotating wheel W, intersecting at its surface at an angle a to form a fringe pattern of spacing δL. This spacing should be too small to be visible with the unaided eye but much greater than the wavelength of light (i.e., between 5 and 50 μm). As it is determined by the beam crossing angle α, you must position BS and M appropriately. The most accurate way to find this angle is to measure the separation of the beams some distance from their intersection; take several such measurements at different positions to obtain an accurate value for α.

The scattered light may be viewed from the side, as shown in Fig. 2. A lens L is used to collect the scattered light and focus it onto the detector D. As before, the detector signal is sent to the spectrum analyzer. The signal frequency of interest may be discerned by blocking either of the interfering beams - it should disappear. Position the wheel transversely to maximize the scattering frequency (i.e., so that the scattering comes from the foremost point on the wheel). Adjust the spectrum analyzer to fully resolve the signal frequency and measure its value. Also measure the diameter of the wheel and estimate its rotation rate observing the detector signal on an oscilloscope - though it may be quite noisy, it should be periodic over one rotation.

Report the measured frequency and its bandwidth. Calculate α and δL, and the rotation rate. How does this compare with the oscilloscope estimate? Which would you expect to be more accurate and why? What are the major sources of error in this experiment, and in particular what is the effect of the bandwidth of the measured scattering frequency?

There are two different ways to explain the presence of the frequency component in the detected scattered light that is related to the rotational speed of the wheel - (i) based on randomly distributed scattering particles moving through the interference pattern on the wheel, (ii) based on the Doppler effect. Please elaborate on this in your lab report.

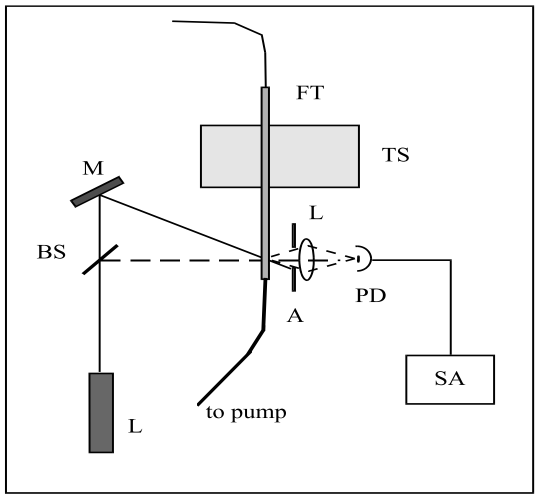

Figure 3: Experimental setup for the flow profile measurements. BS is a microscope slide, FT is the flow tube, and A is an aperture.

Velocity profile of flowing water. The experimental setup is shown in Fig. 3. A small portion of the laser beam is split off by the microscope slide BS (reflectance $\approx 4-5\%$ per face, make sure you block one of the reflected beams). This reference beam is directed through the flow tube FT perpendicular to its faces and through aperture A and lens L onto detector PD. The rest of the laser beam is reflected at mirror M to intersect the reference beam at a point inside FT, at an angle of $10° - 35°$. (This angle is measured as in the transverse velocity measurements above - but be sure to account for the differing indices of water and air.) The tube is filled with water in which a colloidal substance has been suspended - CoffeeMate nondairy creamer works nicely - and is connected to a pump. The suspended particles serve to scatter some of the strong signal beam into the detector where it will be mixed with the reference, any frequency shift will then be observable with the spectrum analyzer. Note the similarity of this arrangement to a Mach-Zehnder interferometer. We should expect the signal beam to be Doppler shifted by an amount determined by the projection of the flow velocity onto the direction of the signal beam (the derivation of the expected shift is left to the student). You should estimate the flow speed and the expected Doppler shift by observing the moving scattering particles.

Several important factors must be considered for this experiment to work properly. First, it is essential that the flow be laminar - a turbulent flow will give Doppler shifts over a wide range of frequencies, blurring the spectrum analyzer peak until it becomes unrecognizable. Second, we must limit the size of the particular scattering region at which we are looking for each measurement - if the volume is large, there will again be a range of frequencies in the spectrum.

This is the function of the lens: it images a small region within the intersection volume onto the detector. Third, we must restrict the range of scattered wavevectors - light scattered off at different angles will have different Doppler shifts, again broadening the frequency peak. This is the function of the aperture A: although the lens will image any light scattered from the volume of interest onto the detector, the aperture blocks all but that light propagating along the direction of the reference beam. (A vertical slit could be used instead. Why?)

Special care must be taken to obtain a signal which is sufficiently clean for taking measurements. The detector should be placed so that it receives the reference beam directly. The position of the lens is particularly critical. Initially determine the approximate height and transverse position by centering the lens on the reference. Then find the longitudinal position in the following manner: Translate the tube so that the beams intersect at the face nearest the detector. Block the beams with a thin sheet of paper and adjust the positions of L and PD to image the beam-spot onto the detector element. Then remove the paper and check that the reference beam is not deflected away from the detector - if it is, repeat the positioning steps. When this procedure is completed, translate the flow tube FT so that the beam intersection is in the middle of the tube. Adjust the spectrum analyzer until a signal is found (start by estimating the expected Doppler shift). The true beat note signal will disappear when either of the beams is blocked. Check to see that the frequency varies with the pump speed.

When you have found the beat note, adjust the position of the various components to maximize the signal amplitude. If the signal is sufficiently above the background (> 10 dB), you will be able to take the necessary data by computer as with the slow wheel. Take measurements at $\approx$1 mm intervals starting at one side and continuing until the other side is reached. In analyzing your data, plot the profile and qualitatively compare it with what would be expected for a laminar flow (look it up). As usual discuss experimental problems, error sources, etc.

4 Summary of Procedures

Upon completion of this experiment you should have sufficient data to obtain the following:

- The rotation rate of the slow wheel using the Doppler effect.
- The speed of the toy train (compare with direct measurement).
- The rotation rate of the fast wheel using the interference-scattering technique.
- The velocity profile of a laminar flow, via the Doppler effect.

References